

# Future Perspectives in Lattice Field Theory\*

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I review some of the contributions which lattice simulations are likely to make during the next five years or so to the development of our understanding of particle physics. Particular emphasis is given to the evaluation of non-perturbative QCD effects in experimentally measurable amplitudes, and the corresponding extraction of fundamental parameters.

## 1. Introduction

I have been asked to talk about the future perspectives of our field, which I find to be an extremely daunting task. There are 388 participants at this conference, each with his or her own view on how the subject may or should develop and all I can do is to present one of these 388 perspectives. The focus of this talk will be on the rôle that lattice simulations are playing now, and will continue to play in the future in the development of particle physics in general and in phenomenology in particular. The most important issue is how well it will be possible to quantify the non-perturbative strong interaction effects in experimentally measurable quantities. This is needed in order to be able to deduce fundamental theoretical information from experimental measurements (e.g. in order to determine the CKM matrix elements from experimental studies of weak decays).

Much of this talk will be based on the deliberations of the panel set up last December by ECFA (the European Committee for Future Accelerators) to advise it on the requirements for high-performance computing for lattice QCD in Europe and I gratefully acknowledge my colleagues on the panel for their insights and help [1]. The panel's terms of reference stated that "the main objective of this study is to assess the high performance computing resources which will be required in the coming years by European physicists working in this field, and to review the scientific opportunities that these resources would open."

Most of the effort in improving the precision of lattice computations of physical quantities will be based on unquenching, the inclusion of light sea-quarks. A key ingredient in attempts to estimate the precision achievable in future simulations is an understanding of the scaling behaviour of the algorithms used to generate full QCD configurations (as the quark masses and lattice size and spacing are varied). I start this talk with a brief review of some recent studies of this question (section 2). The following sections contain some of the main physics questions which will be studied using lattice simulations during the coming years. The rôle of lattice computations in standard model phenomenology will be considered in section 3, and will include a discussion of a selection of physical quantities which are already being computed frequently and also of other quantities for which lattice calculations are just beginning or for which we do not (yet?) understand how to perform the computations, even in principle. Sections 4–6 contain brief discussions of the prospects for lattice simulations of some quantities in hadron spectroscopy, QCD thermodynamics and non-QCD physics. Finally, section 7 contains some closing comments.

Perhaps the main excitement at this conference concerned developments in formulations of chiral fermions on the lattice, and we have had two very interesting plenary talks on this subject [2]. I will not discuss these developments because, although their impact is likely to be very significant, it is still too early to quantify the effect that the new formulations will have on the physics discussed below.

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## 2. Computing Resources and Lattice Parameters

In order to gain some insights into the precision which will be achievable in the next five years or so, it is clearly necessary to try to forecast what computing resources might be available and this has been reviewed by Norman Christ [3]. Once it has been established, that we are aiming at computing power of teraflops, or tens or even hundreds of teraflops, it is also particularly important to have a good understanding of the scaling behaviour of the algorithms used to generate dynamical quark configurations. This is necessary to determine the lattice sizes and spaces and the values of the quark masses with which we will be able to simulate. Our understanding of the scaling behaviour is currently far from complete, and a significant effort will have to be devoted to the study and development of algorithms.

At this conference S. Güsken presented the results of a study based on the performance of algorithms in the Monte-Carlo Runs (at a lattice spacing of 0.08 fm, with 2 flavours of Wilson fermions) of the SESAM/T $\chi$ L collaboration [4]. These authors extrapolate the observed behaviour of their algorithm to larger lattices and lighter quarks and an illustration of their conclusions is given in table 2. The table shows the estimated cost for generating 100 independent configurations in Tflops-months.

Some authors view the estimates in table 2 as being too optimistic. The CP-PACS collaboration have recently tried to *estimate the computer time required for a large-scale full QCD calculation, with the quality of data comparable to that of the present quenched QCD study on the CP-PACS* [5]. They use a renormalisation group improved gauge action and two flavours of degenerate quarks with the clover fermion action and estimate the required time to be of the order of 100 TFlops-year. For example, they estimate that they would require 409 days on a 131 TFlops machine (e.g. on a machine of 4096 PU's with a processing power of 32 GFlops/PU) in order to obtain 25,000 trajectories on a (3 fm)<sup>3</sup> spatial lattice with light quarks with masses down to 15 MeV, for which  $m_\pi/m_\rho \simeq 0.4$  (realized by working on

$\frac{m_\pi}{m_\rho}$	$a = 0.10$	$a = 0.08$	$a = 0.06$
2 fm			
0.60	<b>0.037-0.071</b>	<b>0.20</b>	<b>0.72-1.5</b>
0.50	0.073-0.14	0.38	<b>1.40-3.0</b>
0.40	0.15-0.28	0.77	2.8-6.0
0.30	0.35-0.65	1.8	6.7-14
3 fm			
0.60	<b>0.24-0.45</b>	<b>1.2</b>	<b>4.6-9.7</b>
0.50	<b>0.46-0.86</b>	<b>2.4</b>	<b>8.8-18</b>
0.40	0.94-1.8	4.9	<b>18-38</b>
0.30	2.2-4.1	11	42-89
4 fm			
0.60	<b>0.89-1.7</b>	<b>4.6</b>	<b>16-36</b>
0.50	<b>1.7-3.2</b>	<b>8.8</b>	<b>32-69</b>
0.40	<b>3.5-6.5</b>	<b>18</b>	<b>66-141</b>
0.30	8.2-15	42	<b>152-332</b>

Table 1

Estimated costs for generating 100 independent configurations in Tflops-months [4]. For  $a \neq 0.08$  the two bounds are given to reflect the uncertainty in the behaviour with the lattice spacing. The results from lattices which have an extent of 5 or more pion correlation lengths are printed in bold type.

a  $48^3 \times 96$  lattice with  $a = 0.067$  fm).

A similar study is currently underway in the US, being made as part of the preparation for a proposal to the SSI (Scientific Simulation Initiative), and the preliminary conclusion is also that the Güsken et al. estimate is too optimistic by at least one order of magnitude [6].

In table 2 I present three estimates of time required to generate 1000 independent configurations on a 3 fm lattice, with lattice spacing 0.067 fm and  $m_\pi/m_\rho = 0.4$ . This is essentially the CP-PACS scenario described above. Note that the estimate by Steve Sharpe is for two flavours of staggered fermions, and experience teaches us that the time required for Wilson-like fermions is at least a factor of 10 longer. This discussion demonstrates the importance of further investigations into the scaling behaviour of algorithms. The different groups should start by agreeing on the behaviour in the range of lattice parameters which they are able to simulate. There will nat-

Authors	Estimate	Action
CP-PACS [5]	150 Tflops·Years	RGI, Clover
GLS [4]	13 Tflops·Years	Wilson
Sharpe [41]	2.5 Tflops·Years	Staggered

Table 2

Three estimates of the time required to generate about 1000 independent configurations on a 3 fm lattice, with lattice spacing 0.067 fm and  $m_\pi/m_\rho = 0.4$ .

urally remain uncertainties when extrapolations are made to larger lattices, smaller lattice spacings and lighter values for the quark masses. In spite of the uncertainties, and the obvious expense in computing resources required to perform full QCD simulations with light quarks, the estimates above do indicate that the exciting goal of performing unquenched simulations at the same level of precision as current quenched ones within the next five years or so is a realistic one.

Norman Christ has reviewed the status and prospects for machines for lattice simulations [3]. For the purposes of this talk I will simply assume that at least some of the major collaborations will have access to machines with  $O(10 \text{ TFlops})$  sustained power within the next five years or so.

### 3. Lattice Phenomenology

In this section I will discuss the prospects for quantifying non-perturbative QCD effects for a variety of physical quantities of phenomenological interest. For some of these quantities we understand very well how to perform the computations whereas for many others we do not. In the former category are many quantities which are well measured experimentally, and for which computations are used to demonstrate that lattice systematic uncertainties are under control. This then gives us confidence in the reliability of computations of unknown hadronic matrix elements and other quantities needed to control non-perturbative QCD effects in physical processes.

It has become conventional to present much standard model phenomenology in terms of the unitarity triangle. The unitarity of the CKM-matrix (written in the Wolfenstein parametrisation [7]),

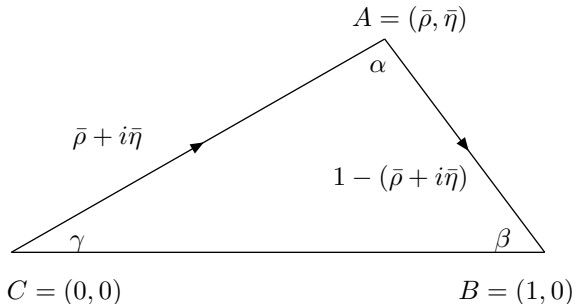


Figure 1. Unitarity Triangle corresponding to the relation in eq.(1).

tion [7]),

$$\begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (1)$$

leads to six unitarity relations, including the one most frequently considered which is shown in Fig. 1 (where  $\bar{\rho} = \rho(1 - \lambda^2/2)$  and  $\bar{\eta} = \eta(1 - \lambda^2/2)$ ). Different physical processes give different loci for the position of the vertex  $A$ . In principle from the intersection of these curves we can determine the position of  $A$ , or if more than two curves do not intersect at the same point, we would deduce that effects of *physics beyond the standard model* are present. In practice, however, hadronic uncertainties are sufficiently large that we arrive at a region of allowed positions for  $A$ . Fig. 2 contains the allowed region deduced from one global analysis of current data [8]. The joint aim of the theoretical and experimental communities is to reduce this region (see below).

The remainder of this section is divided into two parts. In subsection 3.1 I discuss some quantities which are computed frequently in lattice simulations whilst in subsection 3.2 I consider others, for which lattice simulations are either just beginning or for which some conceptual progress is needed before such simulations will be possible.

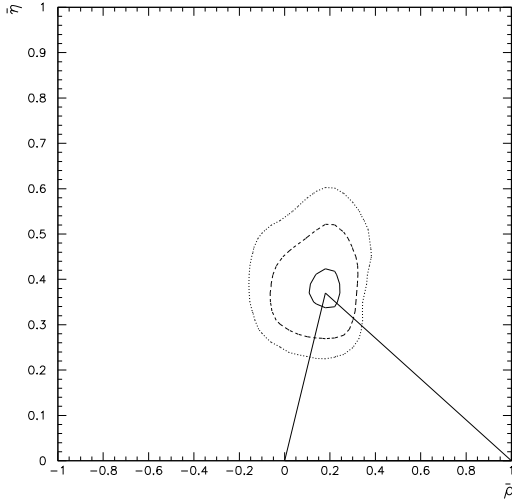


Figure 2. Currently allowed values of the vertex  $A$  of the Unitarity Triangle [8]. The three regions correspond to 5%, 68% and 95% confidence levels.

### 3.1. Frequently Computed Physical Quantities

In table 3 I present our preliminary estimates of the uncertainties in current and future results for a selection of physical quantities and matrix elements which are frequently computed in lattice simulations [1]. The estimates in future calculations are based on being able to generate 1000 configurations at a lattice spacing of 0.08 fm with  $m_\pi/m_\rho = 0.4$  and a sketch of how the improved precision will reduce the allowed region for  $A$  is presented in fig. 3 (without assuming any improvement in the precision of experimental measurements).

The mass of the strange quark,  $m_s$ , is an interesting quantity with which to study the effects of quenching. It has been known for a considerable time that in quenched calculations one obtains a different value for  $m_s$  depending on whether one uses the  $K$ - or  $\Phi$ -mesons [9] (equivalently one cannot reproduce the physical value of the  $J$ -parameter, where  $J$  is basically the slope of  $m_V^2$  vs  $m_{PS}^2$ , where  $V$  ( $PS$ ) represents vector (pseudoscalar) mesons [10]). It will therefore be an important check of our control of full QCD computations to observe whether this problem is cured. At this conference the CP-PACS collaboration pre-

quantity	Present Uncertainty	Future Uncertainty
$\alpha_s$	10%	2%
$m_{u,d,s,c}$	20-25%	5%
$m_b$	5%	1%
$B_K$	15%	5%
Other $B$ 's	15-30%	5%
$f_D, f_{D_s}$	15%	5%
$f_B, f_{B_s}, f_B \sqrt{B_B}$	20%	7%
$F^{D \rightarrow M}(0)$	15-20%	5%
$F^{B \rightarrow M}(0)$	20-35%	7%

Table 3

Estimates of the present and future uncertainties in lattice calculations for a selection of physical quantities, the strong coupling constant, quark masses, the  $B$ -parameter of  $K - \bar{K}$  mixing and other  $B$ -parameters of four-quark operators, the decay constants of heavy mesons and heavy  $\rightarrow$  light form factors at zero momentum transfer.

sented some preliminary results with 2 flavours of sea quarks, indicating that this may be the case (see the talk by R. Mawhinney [11]), and it will be interesting to observe future developments.

Among the recent interesting analyses in standard model phenomenology, Parodi, Roudeau and Stocchi have used the measured values of  $|\epsilon_K|$ ,  $|V_{ub}/V_{cb}|$ ,  $\Delta m_d$  and the bound on  $\Delta m_s$  to obtain  $\bar{\rho} = 0.202 \pm_{-0.059}^{+0.053}$  and  $\bar{\eta} = 0.340 \pm 0.035$  [12] (see also the analysis in ref. [13]), constraining significantly the allowed positions of the vertex  $A$  of the unitarity triangle. These analyses rely on lattice computations of non-perturbative QCD effects, such as those in  $B_K$  (for which the authors use  $B_K = 0.86 \pm 0.09$ ) and  $f_{B_d} \sqrt{B_{B_d}}$  (for which they take  $210 \pm_{-32}^{+39}$  MeV), and of course the reliability of the analyses depends on that of the lattice computations. These authors, however, then perform an interesting exercise, by not including the lattice value of each of the hadronic parameters in turn and instead determining it from the analysis. In this way they find

$$B_K = 0.87 \pm_{-0.20}^{+0.34} \quad \text{and} \quad (2)$$

$$f_{B_d} \sqrt{B_{B_d}} = 223 \pm 13 \text{ MeV} . \quad (3)$$

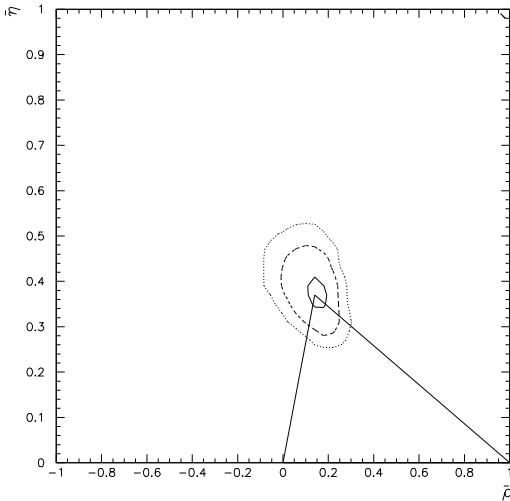


Figure 3. Scenario for the allowed region for the vertex  $A$  of the Unitarity Triangle using the estimates of future errors as in table 3 [8]. The three regions correspond to 5%, 68% and 95% confidence levels.

It is interesting to see the parameters which we are used to calculating in lattice simulations being obtained in this way, albeit relying on lattice values for the remaining matrix elements and assuming no new physics.

One of the decay constants,  $f_{D_s}$ , has been measured directly by several experiments, which the authors of ref. [12] combine to obtain <sup>2</sup>

$$f_{D_s} = 241 \pm 32 \text{ MeV} , \quad (4)$$

in excellent agreement with lattice *predictions*, (e.g. in ref. [15] we quoted  $f_{D_s} = 220 \pm 30$  MeV as the average value from lattice simulations). The experimental result for  $f_{D_s}$  is therefore a significant encouragement to lattice phenomenologists.

Finally in this subsection I consider the form factors for exclusive heavy  $\rightarrow$  light semileptonic and radiative decays, and in particular for  $B \rightarrow \pi, \rho + \text{leptons}$  and  $B \rightarrow K^* \gamma$  decays. The requirement that the  $B$  and final-state light mesons have small momenta (in order to avoid discretisation

<sup>2</sup>Note that the Particle Data Group, more cautiously, present the spread of values from all experiments for  $f_{D_s}$  [14].

errors) implies that direct lattice calculations only yield these form-factors at large values of the momentum transfer (i.e. near the zero-recoil point). The extrapolation of the lattice results to smaller values of the momentum transfer leads to an important source of systematic errors, even though we do have some theoretical guidance from the heavy quark effective theory (HQET), chiral perturbation theory and from axiomatic properties of field theories (such as analyticity and unitarity) in performing these extrapolations [16]. The errors due to the extrapolation will be significantly reduced by increased computing power (allowing us to go to smaller lattice spacings and hence to larger momenta for the mesons). This is an illustration of the fact that for heavy flavour physics, increased computer power may also be fruitfully used to perform larger quenched computations. It is also to be hoped that there will be progress in understanding the scaling behaviour of form-factors at small momentum transfers to make the extrapolations more controllable.

### 3.2. Calculations which are Performed Less Frequently:

The quantities considered in table 3 represent an important set of phenomenologically important parameters, but there are also many other matrix elements and physical quantities for which we need to control the non-perturbative QCD effects. I discuss a selection of these in this subsection.

The major difficulty in lattice phenomenology is that we have no general method for dealing with multihadronic states and final state interactions (see for example ref. [17]). We can compute amplitudes for processes with two particles at rest, which, when combined with chiral perturbation theory proves to be useful for kaon decays (but not for B-decays). I now consider exclusive nonleptonic kaon and B-decays in turn.

#### 3.2.1. $K \rightarrow \pi\pi$ Decays:

$K \rightarrow \pi\pi$  decays were considered in some detail during the early period of lattice phenomenology. As a result of the difficulties which were encountered in the evaluation of the corresponding matrix elements and also because of the develop-

ment of heavy-quark physics the emphasis of lattice simulations changed towards quantities such as those in table 3. More recently the activity in kaon physics has increased again and at this conference was reviewed by Y. Kuramashi [19]. An important difficulty when studying kaon decays using Wilson-like quarks is to control the chiral behaviour (and the corresponding subtraction of power divergences in many cases). The improvement in lattice technology and computing resources, together with theoretical developments such as non-perturbative renormalisation [18] and new formulations for lattice fermions, imply that kaon decays should now become a major area of lattice phenomenology. The motivation for this is further underlined by the new measurements of  $\epsilon'/\epsilon$  from the KTEV and NA48 collaborations [20],

$$\frac{\epsilon'}{\epsilon} = (28 \pm 3.0 \pm 2.8) 10^{-4} \quad [21] \quad (5)$$

$$\frac{\epsilon'}{\epsilon} = (18 \pm 4.5 \pm 5.8) 10^{-4} \quad [22], \quad (6)$$

which are sufficiently large that one might hope to control the expected partial cancellations between the matrix elements of the operators which are conventionally called  $O_6$  and  $O_8$  <sup>3</sup>.

A quantitative understanding of the  $\Delta I = 1/2$  rule in kaon decays is also very desirable and is an important benchmark for lattice QCD computations (note the interesting result in ref. [24], which however has large errors bars, and, as is frequently the case for staggered fermions is plagued by very large perturbative corrections in the matching factors).

### 3.2.2. $B \rightarrow MM$ Decays:

There is a flood of data becoming available on exclusive decays of  $B$ -mesons into two light-mesons, potentially giving fundamental information about the CKM Matrix and CP-violation. This flow of results will increase still further as the  $b$ -factories and other approved experiments

<sup>3</sup>Since the conference the Riken-BNL-Columbia collaboration have presented results from a study using domain wall fermions in which they find surprisingly that these matrix elements have the same sign and obtain a negative result for  $\epsilon'/\epsilon$  ( $\simeq -(12.2 \pm 6.8)10^{-3}$ ) [23].

start taking data. Again the difficulty is in controlling the non-perturbative QCD effects. All the problems of non-leptonic kaon decays are present again, but in addition, the use of chiral perturbation theory is not applicable in  $B$ -decays. New ideas are needed urgently and it may be that it is necessary to combine lattice calculations with models in order to make progress (see for example ref. [25]). Any new ideas can also be tested on the huge amount of experimental data which already exists for nonleptonic charm decays.

The difficulty in controlling hadronic effects in nonleptonic  $B$ -decays serves to underline the beauty of the one *solid-gold* process of  $B \rightarrow J/\Psi K_s$ , which is essentially free of these uncertainties and which will provide an accurate determination of the angle  $\beta$  at the  $b$ -factories <sup>4</sup>.

### 3.2.3. B-Lifetimes:

Lattice calculations of rates for inclusive non-leptonic decays are beginning, and are likely to make an important contribution to standard model phenomenology (see the review talk by Hashimoto [27]). To leading order in the heavy-quark mass all beauty hadrons are predicted to have the same lifetimes, and the corrections to this prediction can be calculated using an expansion in inverse powers of the mass of the  $b$ -quark [28] (see also refs [29] and [30] for reviews and references to the original literature). For the ratio of lifetimes of the  $\Lambda_b$ -baryon and the  $B$ -meson the expansion leads to the prediction:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 1 + 0 - 2\% + O\left(\frac{1}{m_b^3}\right), \quad (7)$$

where the 0 on the right-hand side indicates that there are no operators of dimension 4 which can contribute and the -2% is an estimate of the contribution of dimension 5 operators obtained by comparing the spectroscopy of charmed and beauty mesons and baryons. In view of eq. (7) the experimental measurement,  $\tau(\Lambda_b)/\tau(B_d) = 0.78(4)$  is very puzzling. One possibility is that spectator effects (i.e. effects in which the light quark constituent of the hadrons participate directly in the weak decay process), which in the

<sup>4</sup>The recent measurement from the CDF collaboration gives  $\sin(2\beta) = 0.79^{+0.41}_{-0.44}$  [26].

heavy quark expansion appear at  $O(1/m_b^3)$  but which have a phase-space enhancement [31], are sufficiently large to account for the experimental result. This can be checked in lattice calculations, by computing the matrix elements of the corresponding four-quark operators. A recent exploratory calculation shows that spectator effects are indeed significant, and give a contribution to the right-hand side of eq.(7) of  $-(6-10)\%$  [32,33]. This calculation can readily be improved, and it is important to do so to learn about the practicability of using the heavy quark expansion for predictions of inclusive nonleptonic decays.

A related quantity is  $\Delta\Gamma_s/\Gamma_s$  for which the first lattice results were presented at this conference by the Hiroshima group [34].

### 3.2.4. Lightcone wavefunctions:

Light-cone wave functions contain the non-perturbative QCD effects in hard exclusive processes (such as the form-factors at large momentum transfers [35,36] or nonleptonic  $B$ -decays [37]) and again lattice computations can make a significant contribution to the evaluation of these effects. To my mind, however, there has been insufficient effort in this field. In addition to calculating the moments of these wave-functions in the traditional way, it will also be interesting to explore and develop new suggestions for computing the wave-functions themselves [38].

The main aim of this section was to stress the wide range of QCD phenomenology to which lattice computations can make a significant impact. Of course there are many examples in addition to those presented here, such as deep inelastic structure functions [39], proton decay amplitudes [19], shape functions for inclusive heavy $\rightarrow$ light decays [40], the electric dipole moment of the neutron etc.

## 4. Hadronic Spectroscopy

Hadronic spectroscopy continues to be a benchmark of central importance for lattice computations and the emphasis is now clearly on performing unquenched calculations (see the comprehensive review by Bob Mahwinney [11]). In order to extrapolate the results obtained directly from lat-

Quantity	Uncertainty 2003-5
$m(\eta') - m(\eta)$	30 MeV
$0^{++}$ Mixing	10%
$m(1^{-+})$	20 MeV
$m(H)$	20 MeV

Table 4

Guesstimates of the likely uncertainties which might be achieved for a number of key spectroscopic quantities by 2003-5 or so.

tice simulations to those corresponding to physical  $u$  and  $d$ -quark masses, guidance is needed from chiral perturbation theory. Estimates of how small the masses in simulations need to be in order for chiral perturbation theory to lead to reliable extrapolations are typically in the range corresponding to  $m_\pi/m_\rho \simeq 0.3-0.4$  (see ref. [41] and references therein) and we have seen in section 2 that this is at the limit of what we might hope to achieve in the next few years. A particularly important milestone will have been reached when we are able to observe and control  $\rho \rightarrow \pi\pi$  decays, which given the  $P$ -wave suppression of the decay-rates will take some time.

In table 4 I have presented guesstimates of the relative precision which might be achieved for some key quantities in spectroscopy during the next three to five years or so [1]. A significant effort will be required to get a reasonable handle on all of these quantities. Samples of very large numbers of configurations ( $10^4$  or more) are required to get good signals for flavour singlet mesons. For the exotic  $1^{-+}$  state Doug Toussaint reminded us that the lattice result of 1.9 GeV should be compared with the experimental value of 1.4 GeV, so that we need to study mixing with meson-meson channels [42]. Note that the current theoretical uncertainty on the mass of the  $1^{-+}$  meson is about 200 MeV.

It will also be interesting and important to start to do “nuclear physics” with lattice simulations. These studies will presumably start with the simplest systems, such as the deuteron or the H-dibaryon (bound-state of two  $\Lambda$ -baryons). For these computations to be possible large volumes will be required, but, in addition, the fact that

the binding energies are small,  $O(10 \text{ MeV})$ , implies that again, high statistics will be needed.

## 5. QCD Thermodynamics

The commissioning of the RHIC accelerator at Brookhaven and the approval of the Alice experiment at CERN means that lattice thermodynamics will have a major phenomenological rôle to play as well as a theoretical one. At this conference Fritjof Karsch reminded us that the key features of the quenched theory are well understood (the existence of a first-order phase transition, the equation of state, a critical temperature of about  $270 \text{ MeV}$ ) and that here also the attention is focused on simulations with dynamical quarks. The NUPECC Report on Computational Nuclear Physics concluded that in order to study QCD thermodynamics in simulations with light quarks, a lattice spacing of about  $0.1 \text{ fm}$  and a volume of  $100 \text{ fm}^3$ , about  $10 \text{ Tflops-years}$  of computing effort are required. It appears that the effects of quenching are very significant, for example the critical temperature decreases substantially as quark loops are included ( $T_c \rightarrow 170 \text{ MeV}$  for two sea-quark flavours). In simulations with computing power in the  $10 \text{ Teraflops}$  range, it is expected that it will be possible to determine the critical temperature with a  $5\%$  accuracy.

A schematic diagram of the expected phase structure of QCD in the  $(T, \mu)$  plane is presented in figure 4 (prepared by Simon Hands), where  $\mu$  is the chemical potential. Verification that the structure is indeed as expected, and in particular investigations of the fascinating scenario of a superconducting phase (or phases) of QCD at high  $\mu$  (for a recent review and references to the original literature see ref. [43]), represents a major challenge for the lattice community (see the review by Shuryak at this conference [44]). At non-zero  $\mu$ , the action is complex, and conceptual progress is needed to develop reliable lattice calculations. It is, however, possible to study other quantum field theories with similar expected structures (but for which the action is real) and over the next few years we can look forward to the insights which these simulations will yield.

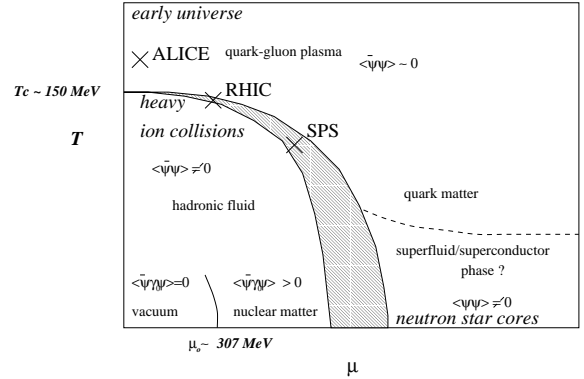


Figure 4. Schematic Phase Diagram for QCD.

## 6. Non-QCD Physics

In this talk I have concentrated on lattice QCD, but we should bear in mind the many application of lattice field theory to other areas of particle physics, some of which we have heard about at this conference. I will now briefly mention applications to the electroweak phase transition and to supersymmetric gauge theories; other applications include the study of phase structures in general (e.g. in QED), other applications in electroweak physics (strong Yukawa sectors, vacuum stability and mass bounds, heavy and strongly interacting Higgs bosons), and studies in quantum gravity. For many non-QCD applications the developments in new formulations of lattice fermions are likely to play a key rôle.

### 6.1. The Electroweak Phase Transition

The motivation for lattice studies of the electroweak phase transition is to gain an understanding of the baryon asymmetry of the universe and this subject has been comprehensively reviewed at this conference by Fodor [45]. For Higgs masses greater than about  $60 \text{ GeV}$ , infrared problems invalidate the use of perturbation theory, requiring the use of lattice simulations. The phase diagram determined from lattice simulations shows a line of 1st-order phase transitions in the  $(T_c, m_H)$  plane, terminating at a 2nd order end-point at  $m_H \simeq 72 \text{ GeV}$ . In order for sphaleron production to be the mechanism for the generation of the observed baryon asymmetry, as expected in the



standard model, there must be a strong first order transition, and hence the mass of the Higgs boson must be less than 72 GeV, in contradiction with the experimental bound of 95 GeV from LEP <sup>5</sup>. It would be reassuring to include the fermions explicitly in these simulations (which would require major computing resources) but already these results present significant evidence that physics beyond the standard model is required.

In the minimal supersymmetric standard model the bound is weaker,  $M_H < 110$  GeV, just above the experimental limit from LEP.

## 6.2. Supersymmetry

Much of the phenomenology of physics “beyond the standard model” is based on supersymmetric field theories. These theories, with their high degree of symmetry, have extremely interesting non-perturbative phenomena (an outstanding example is Seiberg-Witten duality). The subsequent developments have dramatically deepened our understanding of non-perturbative features of gauge theories, although it is not yet clear what the implications for non-supersymmetric theories such as QCD are. This makes the potential investigations of supersymmetric theories using lattice simulations very exciting.

Lattice studies of supersymmetric gauge theories are a formidable challenge and it is natural to start with simpler field theories such as  $N = 1$  supersymmetric Yang-Mills theory and to investigate details of the expected non-perturbative features of confinement and spontaneous chiral symmetry breaking. The computational requirements are similar to those for full QCD and current studies are in their infancy (teaching us for example that the  $SU(2)$  theory has spontaneous discrete chiral symmetry breaking caused by a gaugino condensate). Simulations with small fermion masses are expensive (the supersymmetric limit has vanishing gluino mass), and computing power of the order of a Teraflop sustained would enable most of the important theoretical questions to be answered (mass spectrum, the SUSY potential,

Ward identities). The  $N = 2$  SUSY theory has a considerably more complicated parameter space and so a large amount of effort will be required to explore its phase diagram and so no final results can be expected soon. It remains a very important long-term goal.

## 7. Conclusions

It is my belief that in order for our community to continue to receive support in general, and for state-of-the-art computing facilities in particular, we have to be perceived as having a definite rôle in the development of particle physics. This is certainly the case at present and in this talk I have tried to illustrate the wide range of physical quantities and processes for which lattice results are being used extensively to quantify non-perturbative effects (see also the talk by Mangano [20]). In many cases the leading obstacle to determining standard model parameters or other fundamental information from experimental measurements, is not due to experimental difficulties but to our inability to control the lattice systematic uncertainties as precisely as we would like. In discussing how our computations will improve in the future, it is easy to say that we will do the best that can be done with the resources which will be available. This is probably true, but in my view insufficient, and I believe that we also need to try and predict the “physics-reach” of the next generation (or two) of simulations. No doubt these predictions will have to be revised with time, but we need our theoretical and experimental colleagues to have a realistic picture of our expectations, which are considerable but nevertheless limited. Together with my colleagues on the ECFA panel we have tried to think about the prospects and strategies for the next five years, and some of our preliminary estimates can be found above. I hope that this will be a useful contribution to a debate within the community about the future of lattice field theory.

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<sup>5</sup>72 GeV is the position of the 2nd-order end-point. For a sufficiently strong transition the bound is even stronger,  $m_H < 40$  GeV, even more clearly in contradiction with experiment.

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